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# A Fuzzy Inventory Model For Non-Instantaneous Deteriorating Items Under Conditions Of Permissible Delay In Payments For N-Cycles

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***Abstract:This study develops a fuzzy inventory model to find the minimum relevant inventory cost per unit time for non-instantaneous deteriorating goods over a finite time horizon with exponentially declining demand for n-cycles. The shortages are allowed and back ordered. Under the situation of delay in payments, the inventory model in this study is divided into four cases by the time of shortage, deadline of delay in payment. The holding cost, shortage cost, deterioration cost, purchasing cost and selling price are considered as trapezoidal fuzzy numbers. The signed distance method is used for defuzzification. A numerical example is presented to illustrate the model and the sensitivity analysis is also studied.***

***Key words***

*Inventory, non – instantaneous deteriorating items, exponentially declining demand, delay in payments ,signed distance method, trapezoidal fuzzy number*

## 1.Introduction

The controlling and regulating of deteriorating item is a measure problem in any inventory system. Certain products like vegetables, fruits, electronic components, chemicals deteriorate during their normal storage period. Hence when developing an optimal inventory policy for such products, the loss of inventory due to deterioration cannot be ignored. The researches have continuously modified the deteriorating inventory models so as to more practicable and realistic. In the most inventory models for deteriorating items existed, almost all researches assume that the deterioration of the items in inventory starts from the instant of their arrival in stock. However, in real world, most of goods have a span of maintaining fresh quality of original condition. During that time interval, there is no deterioration. Those items are idled as non-instantaneous items. In the field of inventory control management, it is necessary to consider the inventory problems for non - instantaneous deteriorating items.

Liang-Yuh OUYANG, Kun-shan WU, Mei-chuan CENG [3] were the two earliest researches to consider inventory model for exponential declining demand and during the shortage period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. Later Kai-Wayne chuang, Jing-Fu Lin [1] presented an ordering Quantity model for Non-instantaneous deteriorating items under conditions of permissible delay in payments, they derived for 'n' cycles over a finite planning horizon. Yanlai Liang, Fangning Zhou [9] developed a two warehouse inventory model for deteriorating items under conditionally permissible delay in payment and ZHAO Xian Yu, ZHENG Yi, JIA TAO [10] discussed a ordering policy for Two- phase deteriorating inventory system with changing deterioration rate. Liqun Ji [4] proposed deterministic EOQ inventory model for non-instantaneous deteriorating items with starting and ending without shortages. Sahoo.N.K, Sahoo.C.K [6] considered constant deteriorating items with price dependent demand, time varying holding cost. Recently, Misra.U.K, Sahu.S.K, Bhaskar Bhaula, Raju.L.K[5] provides a detailed review of deteriorating items. They indicated the assumption of Weibull distribution for deteriorating items with permissible delay in payments.

In the traditional inventory EOQ model, the retailer pays for his items as soon as it is received. However in real competitive business world, the supplier offers the retailer a delay period, known as trade credit period. Delay in payment to the supplier is an alternative way of price discount. Hence paying later in directly reduces the purchase cost which attracts the retailers to enhance their ordering quantity. Here Business would earn interest income during this period and pay interest charge after the trade credit period delay in opposite. For retailers, especially small businesses which tend to have a limited number of financing opportunities , rely on trade credit as a source of short-term funds.

In conventional inventory models, various types of uncertainties are classically modeled using the approaches from the probability theory. However, there are uncertainties that cannot be appropriately treated by usual probabilistic models. To define inventory optimization tasks in such environment and to interpret optimal solutions, fuzzy set theory in inventory modeling renders an authenticity to the model formulated since fuzziness is the closest possible approach to reality.

Dutta[2] discussed a fuzzy inventory model without shortages, He used fuzzy trapezoidal numbers for holding cost and ordering cost. Syed [7] developed a fuzzy inventory model without shortages using signed distance method. He also used fuzzy triangular numbers for both ordering cost and holding cost. Umap[8] formed a fuzzy EOQ model for deteriorating items with two warehouses. He considered fuzzy numbers for the parameters such as holding cost and deteriorating cost for two ware houses. He used signed distance method and function principle method for defuzzification of total inventory costs as well as optimum order quantity.

In this paper, a fuzzy inventory model with non-instantaneous deteriorating items is developed for 'n' cycles . We already developed this model for one cycle. We have a

demand function which is exponentially decreasing and the backlogging rate is inversely proportional to the waiting time for the next replenishment. In this study, the inventory model is divided into four cases by the time of shortage and deadline of delay in payment. This study aimed to find the minimum relevant inventory cost per unit time. The holding cost, shortage cost, deterioration cost ,purchasing costs and selling price are considered as trapezoidal fuzzy numbers. The fuzzy total cost is defuzzified using Signed distance method. MATLAB R2007b is used to find the optimal values in numerical examples.

## 2.PRELIMINARIES

***Definition 2.1:***

A fuzzy set $\tilde{a}$ on $R = (-\infty,\infty)$ is called a fuzzy point if its membership function is define by

$$\mu_{\tilde{a}}(x) = \begin{cases} 1, x = a \\ 0, x \neq a \end{cases}$$

Where the point 'a' is called the support of fuzzy set $\tilde{a}$

***Definition 2.2:***

A trapezoidal fuzzy number $\tilde{A} = (a, b, c, d)$ is represented with membership function $\mu_{\tilde{A}}$ is defined as

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, when\ a \le x \le b \\ 1, when\ b \le x \le c \\ \frac{d-x}{d-c}, when\ c \le x \le d \\ 0, otherwise \end{cases}$$

***Definition 2.3:***

A fuzzy set $\tilde{A}$ is defined on R. Then the signed distance of $\tilde{A}$ is defined as

$d(\tilde{A}, 0) = \frac{1}{2}\int_0^1 [A_L(\alpha) + A_R(\alpha)]d\alpha$, where $A_\alpha = [A_L(\alpha), A_R(\alpha)]$

$= [a + (b-a)\alpha, d - (d-c)\alpha], \alpha \in [0,1]$, is the $\alpha$-cut of fuzzy set $\tilde{A}$.

***Definition 2.4:***

Suppose $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers, then arithmetical operations are defined as

***Addition:*** $\tilde{A} + \tilde{B} = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4)$

***Subtraction:***

$\tilde{A} - \tilde{B} = (a_1 - b_1, a_2 - b_2, a_3 - b_3, a_4 - b_4)$

***Multiplication:*** $\tilde{A} \times \tilde{B} = (a_1 b_1, a_2 b_2, a_3 b_3, a_4 b_4)$

***Division:*** $\frac{\tilde{A}}{\tilde{B}} = \left(\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1}\right)$

***Scalar Multiplication*** :For any real constant k,

$$k\tilde{A} = \begin{cases} (ka_1, ka_2, ka_3, ka_4), k \ge 0 \\ (ka_4, ka_3, ka_2, ka_1), k < 0 \end{cases}$$

## 2.Notations and Assumptions

### 2.1 Notations

$I(t)$ = The inventory level at time $t$

$R(t)$ = The demand rate per unit time

A = The initial demand

$\theta$ = The deterioration rate

$\delta$ = The backlogging parameter

n = The number of replenishment cycles

H = The length of planning horizon

T = The length of replenishment cycle(H/n)

M = The permissible delay in settling account

$t_d$ = The length of time with no deterioration ($0 \le t_d \le T$)

k = The length of period with positive inventory ($t_d \le k \le T$)

OC = The fixed cost per order($)

$I_e$ = The interest earned per $ per year

$I_c$ = The interest charged per $ in stocks per year by the supplier

$\tilde{c_h} = (h_1, h_2, h_3, h_4)$ - The fuzzy holding cost per unit per unit time ($)

$\tilde{c_d} = (d_1, d_2, d_3, d_4)$ - The fuzzy deterioration cost per unit per unit time ($)

$\tilde{c_s} = (s_1, s_2, s_3, s_4)$ - The fuzzy shortage cost per unit per unit time ($)

$\tilde{c_p} = (p_1, p_2, p_3, p_4)$ - The fuzzy purchasing cost per unit per unit time ($)

$\tilde{P} = (sp_1, sp_2, sp_3, sp_4)$ - The fuzzy selling price per unit ($)

### 2.2 Assumptions

1. he demand rate is known and decreases exponentially, $R(t) = \begin{cases} Ae^{-\lambda t}, I(t) > 0 \\ D, I(t) \le 0 \end{cases}$, where A(>0) is initial demand and $\lambda, (0 < \lambda < \theta)$ is a constant

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2. The deteriorating rate $\theta$, ($0<\theta<1$) is constant and there is no replenishment or repair of deteriorated units during the period under consideration.
3. The inventory system involves n cycles and the planning horizon is finite.
4. Shortages are allowed except for the last cycle.
5. The backlogging rate is variable during the shortage period and is dependent on the length of the waiting time for the next replenishment. The proportion of customers who would like to accept backlogging at time $t$ is decreasing with the waiting time $(T-t)$ waiting for the next replenishment. To take care of this situation, we have defined the backlogging rate to be $\frac{1}{1+\delta(T-t)}$ when inventory is negative, backlogging parameter $\delta$ is a positive constant.
6. Initializing the cycle, the goods would not deteriorate, but Starting to deteriorate in a constant rate after a fixed period.
7. During the credit period, business can earn the interest income. After this period, business starts paying for interest charges.

## 3. Model formulation

The objective of the inventory problem is to find the total relevant cost as low as possible. The behavior of inventory system at any time is depicted in figure.

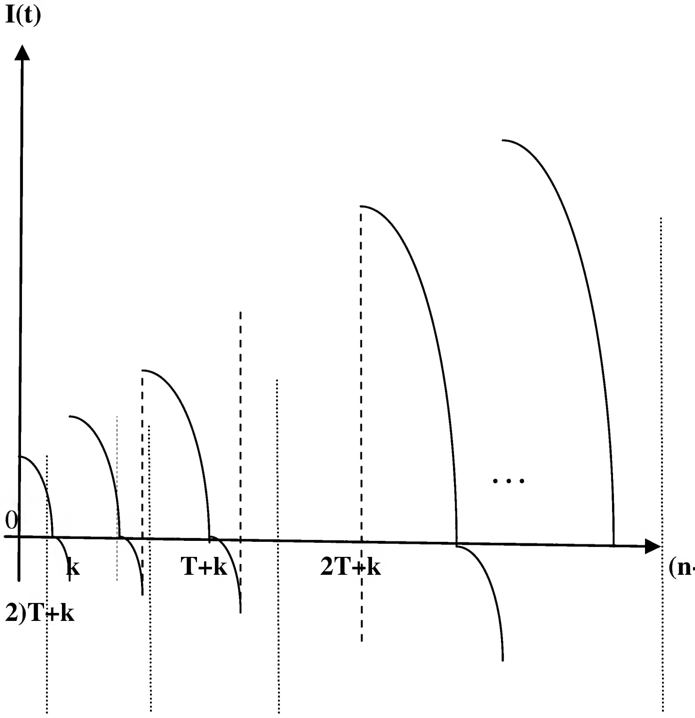


**Figure 1**

**Inventory model for 'n' cycles**

This inventory model has 'n' cycles over a finite planning horizon, and every cycle is beginning at $(j-1)T$, j=1,2,…,n. This study assumed that the goods would start to deteriorate in a constant rate after time $(j-1)T+t_d$ and shortage occurs during the period $(j-1)T+k$ to $jT$, j=1,2,…,n. We aimed to find the optimal value of k over the period $[0,H]$ for finding the optimal cost in model.

The change of inventory level can be described by the following equations

$$\frac{dI(t)}{dt}=-Ae^{-\lambda t},(j-1)T<t<(j-1)T+t_d$$

$$\frac{dI(t)}{dt}+\theta I(t)=-Ae^{-\lambda t},(j-1)T+t_d<t<(j-1)T+k$$

$$\frac{dI(t)}{dt}=\frac{-D}{1+\delta(jT-t)},(j-1)T+k<t<jT$$

with boundary conditions
$$I[(j-1)T]=I_{max}(j),I[(j-1)T+k]=0 \quad (4)$$

The solutions of equations (1) – (3) are given by

$$I(t)=I_{max}(j)+\frac{A}{\lambda}\left[e^{-\lambda t}-e^{-\lambda(j-1)T}\right],(j-1)T<t<(j-1)T+t_d$$

$$I(t)=\frac{A}{(\lambda-\theta)e^{\theta t}}\left[e^{-(\lambda-\theta)t}-e^{-(\lambda-\theta)[(j-1)T+k]}\right],(j-1)T+t_d<t<($$

$$I(t)=\frac{D}{\delta}\log\left[\frac{1+\delta(jT-t)}{1+\delta(jT-(j-1)T-k}\right],(j-1)T+k<t<jT$$

At $t=(j-1)T+t_d$,

$$I_{max}(j)=\frac{A}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}}\left[e^{-(\lambda-\theta)[(j-1)T+t_d]}-e^{-(\lambda-\theta)[(j-1)T+k]}\right]-\frac{A}{\lambda}\left[e^{-\lambda[(j-1)T+t_d]}-e^{-\lambda(j-1)T}\right]$$


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The change of inventory level in the last cycle can be described by the following equation

$$I(t) = I_{max}(n) + \frac{A}{\lambda}\left[e^{-\lambda t} - e^{-\lambda(n-1)T}\right], (n-1)T < t < (n-1)T + t_d$$

$$I(t) = \frac{A}{e^{\theta t}(\lambda-\theta)}\left[e^{-(\lambda-\theta)t} - e^{-(\lambda-\theta)H}\right], (n-1)T + t_d < t < H$$

At $t = (n-1)T + t_d$ ,

$$I_{max}(n) = \frac{A}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}}\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right] - \frac{A}{\lambda}\left[e^{-\lambda[(n-1)T+t_d]} - e^{-\lambda(n-1)T}\right]$$

The fuzzy holding cost($\widetilde{HC}$) is given by

$$= \tilde{c_h}\left[\sum_{j=1}^{n-1}\left\{\int_{(j-1)T}^{(j-1)T+t_d} I(t)\,dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I(t)\,dt\right\} + \int_{(n-1)T}^{(n-1)T+t_d} I(t)\,dt + \int_{(n-1)T+t_d}^{H} I(t)\,dt\right]$$

$$= \tilde{c_h}\left[\sum_{j=1}^{n-1}\left\{\frac{At_d\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{At_d e^{-\lambda[(j-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(j-1)T+t_d]} - e^{-\lambda[(j-1)T]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\lambda[(j-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-[(\lambda-\theta)[(j-1)T+k]+\theta[(j-1)T+t_d]]}\right]}{\theta(\lambda-\theta)}\right\} + \frac{At_d\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}} - \frac{At_d e^{-\lambda(n-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(n-1)T+t_d]} - e^{-\lambda[(n-1)T]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda H} - e^{-[(\lambda-\theta)H+\theta[(n-1)T+t_d]]}\right]}{\theta(\lambda-\theta)}\right]$$

$$= (h_1, h_2, h_3, h_4)\left[\sum_{j=1}^{n-1}\left\{\frac{At_d\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{At_d e^{-\lambda[(j-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(j-1)T+t_d]} - e^{-\lambda[(j-1)T]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\lambda[(j-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-[(\lambda-\theta)[(j-1)T+k]+\theta[(j-1)T+t_d]]}\right]}{\theta(\lambda-\theta)}\right\} + \frac{At_d\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}} - \frac{At_d e^{-\lambda(n-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(n-1)T+t_d]} - e^{-\lambda[(n-1)T]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda H} - e^{-[(\lambda-\theta)H+\theta[(n-1)T+t_d]]}\right]}{\theta(\lambda-\theta)}\right]$$

(12)

The fuzzy deterioration cost($\widetilde{DC}$) is given by

$$= \tilde{c}_d\left[\sum_{j=1}^{n-1}\left\{I_{max}(j) - \int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}\,dt\right\} + I_{max}(n) - \int_{(n-1)T+t_d}^{H} I(t)\,dt\right]$$

(10)

$$= \tilde{c}_d\left[\sum_{j=1}^{n-1}\left\{\frac{A\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{Ae^{-\lambda[(j-1)T+t_d]}}{\lambda} + \right. \right.$$

$$+ \frac{A\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}} - \frac{Ae^{-\lambda[(n-1)T+t_d]}}{\lambda} +$$

(11)

$$= (d_1, d_2, d_3, d_4)\left[\sum_{j=1}^{n-1}\left\{\frac{A\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{Ae^{-\lambda[}}{} \right. \right.$$

$$\frac{A\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}} - \frac{Ae^{-\lambda[(n-1)T+t_d]}}{\lambda} +$$

(13)

The fuzzy shortage cost ($\widetilde{SC}$) is given by

$$= \tilde{c}_s\left[\sum_{j=1}^{n-1}\int_{(j-1)T+k}^{jT} I(t)\,dt\right]$$

$$= -\frac{\tilde{c}_s D(n-1)}{\delta}\left[1 + (1+\delta(T-k))\{log(1+\delta(T-k)) - 1\} + (T-k\right.$$

$$= -\frac{(s_1, s_2, s_3, s_4)D(n-1)}{\delta}\left[1 + (1+\delta(T-k))\{log(1+\delta(T-k)) - \right. + (T-k)log(1+\delta(T-k))\Big]$$

**CASE I ($0 \le M \le t_d$)**

The length of delay in payment (M) is absolutely is less than the length with no deterioration ($t_d$) in this case. Business can earn the interest income during credit period. However, business starts paying for interest charges after the credit period.

The fuzzy total interest expenditure($\widetilde{IC}$)

$$= \tilde{c}_p I_c\left[\sum_{j=1}^{n-1}\left\{\int_{(j-1)T+M}^{(j-1)T+t_d} I(t)\,dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I(t)\,dt\right\} + \int_{(n-1)T+M}^{(n-1)T+t_d} I(t)\,dt + \right.$$

$$= (p_1, p_2, p_3, p_4) I_c \left[ \sum_{j=1}^{n-1} \left\{ \int_{(j-1)T+M}^{(j-1)T+t_d} I(t)\,dt + \int_{(j-1)T+t_d}^{(j-1)T+k} I(t)\,dt \right\} + \int_{(n-1)T+M}^{(n-1)T+t_d} I(t)\,dt + \int_{(n-1)T+t_d}^{H} I(t)\,dt \right]$$

$$= (p_1, p_2, p_3, p_4) I_c \left[ \sum_{j=1}^{n-1} \left\{ \frac{A(t_d - M)\left[e^{-(\lambda-\theta)[(j-1)T+t_d]} - e^{-(\lambda-\theta)[(j-1)T+k]}\right]}{(\lambda-\theta)e^{\theta[(j-1)T+t_d]}} - \frac{A(t_d - M)e^{-\lambda[(j-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(j-1)T+t_d]} - e^{-\lambda[(j-1)T+M]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\lambda[(j-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-[(\lambda-\theta)[(j-1)T+k]+\theta[(j-1)T+t_d]]}\right]}{\theta(\lambda-\theta)} \right\} + \frac{A(t_d - M)\left[e^{-(\lambda-\theta)[(n-1)T+t_d]} - e^{-(\lambda-\theta)H}\right]}{(\lambda-\theta)e^{\theta[(n-1)T+t_d]}} - \frac{A(t_d - M)e^{-\lambda[(n-1)T+t_d]}}{\lambda} - \frac{A\left[e^{-\lambda[(n-1)T+t_d]} - e^{-\lambda[(n-1)T+M]}\right]}{\lambda^2} - \frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+t_d]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda H} - e^{-[(\lambda-\theta)H+\theta[(n-1)T+t_d]]}\right]}{\theta(\lambda-\theta)} \right]$$

The total interest revenue($\widetilde{IE}$)

$$= \tilde{P} I_e \sum_{j=1}^{n} \int_{(j-1)T}^{(j-1)T+M} A e^{-\lambda t}[t - (j-1)T]\,dt$$

$$= (sp_1, sp_2, sp_3, sp_4) I_e \sum_{j=1}^{n} \frac{A e^{-\lambda(j-1)T}}{\lambda^2}\left[1 - \lambda M e^{-\lambda M} - e^{-\lambda M}\right]$$

Therefore here

$$\widetilde{TC} = \widetilde{HC} + \widetilde{DC} - \widetilde{SC} + \widetilde{IC} - \widetilde{IE} + n\,OC$$

**CASE II $(t_d \le M \le k)$**

Here the period of delay in payment (M) is more length than the period with no deterioration $(t_d)$ but less than length of period with positive inventory (k)

The fuzzy total interest expenditure($\widetilde{IC}$)

$$= (p_1, p_2, p_3, p_4) I_c \left[ \sum_{j=1}^{n-1} \left\{ \int_{(j-1)T+M}^{(j-1)T+k} I(t)\,dt \right\} + \int_{(n-1)T+M}^{H} I(t)\,dt \right]$$

$$= (p_1, p_2, p_3, p_4) I_c \left[ \sum_{j=1}^{n-1} \left\{ - \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-\lambda[(j-1)T+M]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda[(j-1)T+k]} - e^{-[(\lambda-\theta)[(j-1)T+k]+\theta[(j-1)T+M]]}\right]}{\theta(\lambda-\theta)} \right\} - \ldots + \frac{A\left[e^{-\lambda H} - e^{-[(\lambda-\theta)H+\theta[(n-1)T+M]]}\right]}{\theta(\lambda-\theta)} \right]$$

Thefuzzy total interest revenue($\widetilde{IE}$)

$$= \tilde{P} I_e \sum_{j=1}^{n} \int_{(j-1)T}^{(j-1)T+M} A e^{-\lambda t}[t - (j-1)T]\,dt$$

$$= (sp_1, sp_2, sp_3, sp_4) I_e \sum_{j=1}^{n} \frac{A e^{-\lambda(j-1)T}}{\lambda^2}\left[1 - \lambda M e^{-\lambda M} - e^{-\lambda M}\right]$$

Therefore here

$$\widetilde{TC} = \widetilde{HC} + \widetilde{DC} - \widetilde{SC} + \widetilde{IC} - \widetilde{IE} + n\,OC \qquad (15)$$

**CASE III $(k \le M \le T)$**

The period of delay in payment(M) is more length than period with positive inventory (k) except for the last cycle. Therefore, the interest charge occurs in the last cycle only.

The fuzzy total interest expenditure($\widetilde{IC}$)

$$= \tilde{c}_p I_c \left[ \int_{(n-1)T+M}^{H} I(t)\,dt \right] \qquad (16)$$

$$= (p_1, p_2, p_3, p_4) I_c \left[ - \frac{A\left[e^{-\lambda H} - e^{-\lambda[(n-1)T+M]}\right]}{\lambda(\lambda-\theta)} + \frac{A\left[e^{-\lambda H} - e^{-[(\lambda-\theta)H+\theta[(n-1)T+M]]}\right]}{\theta(\lambda-\theta)} \right] \qquad (17)$$

The total interest revenue($\widetilde{IE}$)

$$= \tilde{P} I_e \left[ \sum_{j=1}^{n} \left\{ \int_{(j-1)T}^{(j-1)T+k} A e^{-\lambda t}[t - (j-1)T]\,dt + (M-k) \int_{(j-1)T}^{(j-1)T+k} A e^{-\lambda t}\,dt \ldots \right. + \int_{(n-1)T}^{(n-1)T+M} A e^{-\lambda t}(t - (n-1)T)\,dt \right]$$

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$$= (sp_1, sp_2, sp_3, sp_4) I_e \left[ \sum_{j=1}^{n} \frac{Ae^{-\lambda(j-1)T}}{\lambda^2} \left[1 - e^{-\lambda k} - \lambda M e^{-\lambda k} + (M-k)\lambda\right] + \frac{Ae^{-\lambda(n-1)T}}{\lambda^2}\left[1 - \lambda M e^{-\lambda M} - e^{-\lambda M}\right] \right] \quad (22)$$

Therefore here

$$\widetilde{TC} = \widetilde{HC} + \widetilde{DC} - \widetilde{SC} + \widetilde{IC} - \widetilde{IE} + n\,OC \quad (23)$$

**CASE IV $(T \le M)$**

The period of delay in payment(M) is absolutely more length than a cycle(T) . To compare with case III, there is no interest charge in this case.

The fuzzy total interest revenue$(\widetilde{IE})$

$$= \tilde{P} I_e \left[ \sum_{j=1}^{n} \left\{ \int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}[t-(j-1)T]dt + (M-k)\int_{(j-1)T}^{(j-1)T+k} Ae^{-\lambda t}dt \right\} + \int_{(n-1)T}^{H} Ae^{-\lambda t}(t-(n-1)T)dt + (M-k)\int_{(n-1)T}^{H} Ae^{-\lambda t}dt \right]$$

$$= (p_1, p_2, p_3, p_4) I_e \left[ \sum_{j=1}^{n} \frac{Ae^{-\lambda(j-1)T}}{\lambda^2}\left[1 - e^{-\lambda k} - \lambda M e^{-\lambda k} + (M-k)\lambda\right] + \frac{Ae^{-\lambda(n-1)T}}{\lambda^2}\left[1 + \lambda(M-T)\right] - \frac{Ae^{-\lambda H}}{\lambda^2}\left[1 + \lambda[H-(n-1)T] + \lambda(M-T)\right] \right] \quad (24)$$

Therefore here

$$\widetilde{TC} = \widetilde{HC} + \widetilde{DC} - \widetilde{SC} - \widetilde{IE} + n\,OC \quad (25)$$

The fuzzy total cost is defuzzified using signed distance method

For a minimum $d(\widetilde{TC})$, $\frac{\partial d(\widetilde{TC})}{\partial k} = 0 \; and \; \frac{\partial^2 d(\widetilde{TC})}{\partial k^2} > 0$ in all the cases.

**4.NUMERICAL EXAMPLE**

Consider the following data A=2, $\lambda = 0.01, \theta = 0.1$, $t_d = \frac{60}{365}$, $\delta = 1$, OC=10,H=3,n=3,D=5,$c_h$ =(2.6,2.8,3.2,3.4), $c_d = (6.6,6.8,7.2,7.4)$, $c_s = (5.6,5.8,6.2,6.4)$, $c_p = (7.6,7.8,8.2,8.4)$, P=(11.6,11.8,12.2,12.4), $I_e = 0.03$, $I_c = 0.08$

| CASE | M | k | TOTAL COST |
|---|---|---|---|
| CASE I $(0 \le M \le t_d)$ | 30/365 | 0.925287 | \$43.79 |
| CASE II $(t_d \le M \le k)$ | 90/365 | 0.926305 | \$41.95 |
| CASE III $(k \le M \le T)$ | 360/365 | 0.935610 | \$40.34 |
| CASE IV $(T \le M)$ | 375/365 | 0.935646 | \$40.26 |

**5.Conclusion**

In this paper, a fuzzy inventory model for non – instantaneous deteriorating items with permissible delay in payment over a finite horizon. With the reason of permissible delay in payments and non-instantaneous deterioration, the total is minimized compared to existing models. The holding cost, shortage cost, deterioration cost, purchasing cost, selling price are considered as trapezoidal fuzzy numbers. Signed distance method is used for defuzzification. MATLAB R2007b is used to find the optimal values in numerical examples.

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